PM Equation

The hourly Penman-Monteith equation for estimating reference evapotranspiration used on the CIMIS computer is a version described in the Food and Agricultural Organization's Irrigation and Drainage Paper No. 56 (Allen, R.G., L.S. Pereira, D. Raes, and M. Smith. 1998). However, bulk surface resistance is adapted from the ASCE Task Committee on Standardization of Reference Evapotranspiraton (2000). Two reference evapotranspiration equations are employed, one for a grass reference and the other for alfalfa.

Reference Evapotranspiration for Grass

$$ET_o = \frac{\Delta(R_n - G)}{\lambda[\Delta + \gamma(1 + C_d u_2)]} + \frac{\gamma \frac{37}{T_a + 273.16} u_2 (e_s - e_a)}{\Delta + \gamma(1 + C_d u_2)}$$ (EQ1)

Where

- ETo = grass reference evapotranspiration (mm h^{-1})
- Δ = slope of saturation vapor pressure curve (kPa $°C^{-1}$) at mean air temperature (*T*)
- R_n = net radiation (MJ m^{-2} h^{-1})
- G = soil heat flux density (MJ m^{-2} h^{-1})
- γ = psychrometric constant (kPa$°C^{-1}$)
- T_a = mean hourly air temperature (°C)
- U_2 = wind speed at 2 meters (m s^{-1})
- e_s = saturation vapor pressure (kPa) at the mean hourly air temperature (T) in °C
- e_a = actual vapor pressure (kPa) at the mean hourly air temperature (T) in °C
- *lamda* = latent heat of vaporization (MJ kg^{-1})
- C_d = bulk surface resistance and aerodynamic resistance coefficient

- When R_n>0 (daytime)

  Soil heat flux (MJ m^{-2} h^{-1})

  $G = 0.1 R_n$ (EQ2)

  Bulk surface resistance and aerodynamic resistance coefficient

  $C_d = 0.24$ (EQ3)

- When R_n <= 0 (nighttime)

  Soil heat flux (MJ m^{-2} h^{-1})

  $G = 0.5 R_n$ (EQ4)

  Bulk surface resistance and aerodynamic resistance coefficient

  $C_d = 0.96$ (EQ5)

Reference Evapotranspiration for Alfalfa

$$ET_r = \frac{\Delta(R_n - G)}{\lambda[\Delta + \gamma(1 + C_d u_2)]} + \frac{\gamma \frac{66}{T_a + 273.16} u_2 (e_s - e_a)}{\Delta + \gamma(1 + C_d u_2)}$$ (EQ6)

Where

- ETr = alfalfa reference evapotranspiration (mm h^{-1})
- Δ = slope of saturation vapor pressure curve (kPa °C^{-1}) at mean air temperature (*T*)
- R_n = net radiation (MJ m^{-2} h^{-1})
- G = soil heat flux density (MJ m^{-2} h^{-1})
- γ = psychrometric constant (kPa °C^{-1})
- T_a = mean hourly air temperature (°C)
- U_2 = wind speed at 2 meters (m s^{-1})
- e_s = saturation vapor pressure (kPa) at the mean hourly air temperature (T) in °C
- e_a = actual vapor pressure (kPa) at the mean hourly air temperature (T) in °C
- *lamda* = latent heat of vaporization in (MJ kg^{-1})
- C_d = bulk surface resistance and aerodynamic resistance coefficient

- When R_n >0 (daytime)

  Soil heat flux (MJ m^{-2} h^{-1})

  $$G = 0.04\,R_n \quad \text{(EQ7)}$$

  Bulk surface resistance and aerodynamic resistance coefficient

  $$C_d = 0.25 \quad \text{(EQ8)}$$

- When R_n <= 0 (nighttime)

  Soil heat flux (MJ m^{-2} h^{-1})

  $$G = 0.2\,R_n \quad \text{(EQ9)}$$

  Bulk surface resistance and aerodynamic resistance coefficient

  $$C_d = 1.70 \quad \text{(EQ10)}$$

Other components in EQ1 and EQ6 are calculated as follows:

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i. Delta

$$\Delta = \frac{4099\,e_s}{(T_a + 237.3)^2} \quad \text{(EQ11)}$$

T_a = mean hourly air temperature (°C)

ii. Net radiation (MJ m^{-2} h^{-1})

$$R_n = 0.77\,R_s - R_{nl} \quad \text{(EQ12)}$$

R_s = solar radiation (MJ m^{-2} h^{-1})
R_{nl} = net longwave radiation (MJ m^{-2} h^{-1})

iii. Psychrometric constant

$$\gamma = 0.00163 \frac{P}{\lambda} \quad \text{(EQ13)}$$

P = barometric pressure (kPa)

$$P = 101.3\left[\frac{293 - 0.0065\,A}{293}\right]^{5.26} \quad \text{(EQ14)}$$

A = station elevation above mean sea level (meters)

iv. Saturation vapor pressure (kPa)

$$e_s = 0.6108 \exp\left[\frac{17.27\,T_a}{T_a + 237.3}\right] \quad \text{(EQ15)}$$

T_a = mean hourly air temperature (°C)

v. Actual vapor pressure (kPa)

$$e_a = e_s * \frac{RH}{100} \quad \text{(EQ16)}$$

e_s = saturation vapor pressure (kPa)
RH = relative humidity (%)

vi. Latent heat of vaporization (MJ kg^{-1})

$$\lambda = 2.501 - 0.002361\,T_a \quad \text{(EQ17)}$$

T_a = mean hourly air temperature (°C)

Extraterrestrial Radiation Equation

To calculate net longwave radiation in Eq. 10, extraterrestrial radiation, R_a, is required. The calculation of R_a is based on Duffie and Beckman (1991).

$$R_a = \frac{12(60)}{\pi} G_{sc}\, d_r\, *$$

$$[(\omega_2 - \omega_1)\sin\phi\sin\delta + \cos\phi\cos\delta(\sin(\omega_2) - \sin(\omega_1))] \quad \text{(EQ18)}$$

- G_{SC} = solar constant (0.082 MJ m^{-2} min^{-1})
- d_r = inverse relative distance Earth-Sun (correction for eccentricity of Earth's orbit around the sun)
- ω_1 = Solar time angle 1/2 hour before ω, that is, at the beginning of period (radians)

- ω_2 = Solar time angle 1/2 hour after ω, at end of period (radians)
- fe = Station latitude (radians)
- δ = Declination of the sun above the celestial equator (radians)

Where

$$d_r = 1 + 0.033 \cos\left[\frac{2\pi}{365} J\right]$$ (EQ19)

J = day of year (1-366)

$$\omega_1 = \omega - \left[\frac{1}{2}\right]\left[\frac{\pi}{12}\right]$$ (EQ20)

$$\omega_2 = \omega + \left[\frac{1}{2}\right]\left[\frac{\pi}{12}\right]$$ (EQ21)

- **Solar time angle ω at midpoint of the hourly period is given by:**

  $$\omega = \frac{\pi}{12}\left[\left(\left(t - 0.5\right) - \frac{4}{60}\left(L_m - L_z\right) + S_c\right) - 12\right]$$ (EQ22)

  t = standard clock time (1-24)

  L_m = longitude of measurement location (weather station) in degrees

  L_z = longitude of the local time meridian (degrees West), 120° for Pacific time zone

  $$S_c = 0.1645 \sin(2b) - 0.1255 \cos(b) - 0.025 \sin(b)$$ (EQ23)

  $$b = \frac{2\pi}{364}(J - 81)$$ (EQ24)

  J = day of year (1-366)

  $$\phi = \frac{\pi Y}{180}$$ (EQ25)

  Y = latitude (decimal degrees)

  $$\delta = 0.409 \sin\left[\frac{2\pi}{365} J - 1.39\right]$$ (EQ26)

- **Long wave Radiation Equation (MJ m^{-2} h^{-1})**

  $$R_{nl} = f\,\epsilon\,\sigma\left(T_{ak}\right)^4$$ (EQ27)

  f = cloudiness factor

  epsilon = net emissivity of the surface

σ = Stefan-Boltzmann constant ($2.04*10^{-10}$) MJ m^{-2} h^{-1} K^{-4}

Ta_k = mean hourly temperature (Kelvin)

For daytime hours defined as θ >10°

$$f = 1.35 \frac{R_s}{R_{so}} - 0.35$$ (EQ28)

R_s = solar radiation (MJ m^{-2} h^{-1})

R_{so} = clear sky total global radiation at the earth surface (MJ m^{-2} h^{-1})

For nighttime hours, θ <= 10 °

$$f = 1.35 \frac{R_{s\,pm}}{R_{so\,pm}} - 0.35$$ (EQ29)

R_{spm} = solar radiation (MJm^{-2} h^{-1}) during the last hour for which θ>10 °

R_{sopm} = clear sky solar radiation (MJ $m^{-2}h^{-1}$) during the last hour for which θ>10 °

When f<0 set f = 0.595, and when f>1.0 set f=1.0

$$R_{so} = R_a\,(\,0.75 + 2.00 * 10^{-5} A\,)$$ (EQ30)

A = station elevation above mean sea level (meters)

$$\varepsilon' = 0.34 - 0.14\sqrt{e_a}$$ (EQ31)

Solar altitude is calculated as:

$$\theta = arcsin\,[\,\sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega\,] * \frac{180}{\pi}$$ (EQ32)

All terms have been defined above.

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